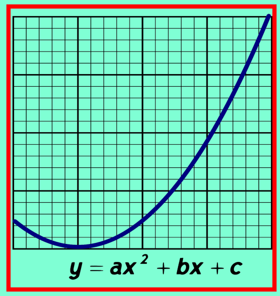
Math 125

Fall 2021

Lecture 44



Class QZ 33

Solve

$$\begin{cases} x + y = 1 & x = 1 - y \\ x^2 + xy - y^2 = -5 \end{cases}$$

Hint: Isolate one Variable, use Subs. Method. Final Ans in Ordered-Pairs

$$(1-y)^2 + y(1-y) - y^2 = -5$$

$$(1-y)(1-y) + y - y^2 - y^2 = -5$$

$$1 - y \cancel{-y} \cancel{+y^2} \cancel{+y} \cancel{-y^2} - y^2 = -5$$

$$1 - y - y^2 = -5$$

$$y^2 + y - 1 - 5 = 0$$

$$\boxed{y^2 + y - 6 = 0}$$

$$(y+3)(y-2) = 0$$

$$y = -3 \qquad y = 2$$

$$x = 1 - y$$

$$x = 1 - (-3) \qquad x = 4$$

$$x = 1 - 2 \qquad x = -1$$

$$\boxed{\{(4,-3), (-1,2)\}}$$

Ex.:

Solve

$$\begin{cases} \frac{3}{x^2} + \frac{1}{y^2} = 7 \\ \frac{5}{x^2} - \frac{2}{y^2} = -3 \end{cases}$$

Notice

$\frac{3}{x^2} = 3\left(\frac{1}{x^2}\right)$

$\frac{5}{x^2} = 5\left(\frac{1}{x^2}\right)$

$\frac{2}{y^2} = 2\left(\frac{1}{y^2}\right)$

$$\begin{cases} 3\left(\frac{1}{x^2}\right) + \frac{1}{y^2} = 7 \\ 5\left(\frac{1}{x^2}\right) - 2\left(\frac{1}{y^2}\right) = -3 \end{cases}$$

Let $\frac{1}{x^2} = A$, $\frac{1}{y^2} = B$

$$2\begin{cases} 3A + B = 7 \\ 5A - 2B = -3 \end{cases} \Rightarrow \begin{cases} 6A + 2B = 14 \\ 5A - 2B = -3 \end{cases}$$

$11A = 11$

$\boxed{A = 1}$

$3(1) + B = 7$

$\boxed{B = 4}$

$\frac{1}{x^2} = A$

$\frac{1}{x^2} = 1$

$x^2 = 1$

$\boxed{x = \pm 1}$

$\frac{1}{y^2} = B$

$\frac{1}{y^2} = 4$

$4y^2 = 1$

$y^2 = \frac{1}{4}$

$\boxed{y = \pm \frac{1}{2}}$

Final Ans:

$\left(1, \frac{1}{2}\right), \left(1, \frac{-1}{2}\right)$

$\left(-1, \frac{1}{2}\right), \left(-1, \frac{-1}{2}\right)$

Solve

$$\begin{cases} \dfrac{2}{x^2} + \dfrac{1}{y^2} = 11 \\ \dfrac{4}{x^2} - \dfrac{2}{y^2} = -14 \end{cases}$$

Let $A = \dfrac{1}{x^2}$, $B = \dfrac{1}{y^2}$

$$2\begin{cases} 2A + B = 11 \\ 4A - 2B = -14 \end{cases} \Rightarrow \begin{cases} 4A + 2B = 22 \\ 4A - 2B = -14 \end{cases}$$

$8A = 8$

$\boxed{A = 1}$

$2(1) + B = 11$

$\boxed{B = 9}$

$\dfrac{1}{x^2} = 1$ $\quad$ $\dfrac{1}{y^2} = 9$

$x^2 = 1$ $\quad$ $y^2 = \dfrac{1}{9}$

$\boxed{x = \pm 1}$ $\quad$ $\boxed{y = \pm \frac{1}{3}}$

Final Ans

$(1, \frac{1}{3}), (1, -\frac{1}{3}),$
$(-1, \frac{1}{3}), (-1, -\frac{1}{3})$

Sum of squares of two numbers is 10.
their Product is 3. x, y
find all such numbers.

$$\begin{cases} x^2 + y^2 = 10 \\ xy = 3 \end{cases} \rightarrow y = \frac{3}{x}$$

$x^2 + \left(\frac{3}{x}\right)^2 = 10$

$x^2 + \dfrac{9}{x^2} = 10$

multiply by x^2 to clear fractions

$x^2 \cdot x^2 + \cancel{x^2} \cdot \dfrac{9}{\cancel{x^2}} = x^2 \cdot 10$

$x^4 + 9 = 10x^2$

$x^4 - 10x^2 + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

$x^2 - 9 = 0$	$x^2 - 1 = 0$
$x^2 = 9$	$x^2 = 1$
$x = \pm 3$	$x = \pm 1$
$y = \pm 1$	$y = \pm 3$

Here are Possible choices

3,1 $\quad$ -3,-1 $\quad$ 1,3 $\quad$ -1,-3

The product of two numbers is 4.

The difference of their squares is 15.

Find all such numbers.

$$\begin{cases} xy = 4 \\ x^2 - y^2 = 15 \end{cases}$$

$xy = 4 \rightarrow y = \frac{4}{x}$

$x^2 - \left(\frac{4}{x}\right)^2 = 15$

$\rightarrow x^2 - \frac{16}{x^2} = 15$

Multiply by x^2

$x^4 - 16 = 15x^2$

$\rightarrow x^4 - 15x^2 - 16 = 0$

$(x^2+1)(x^2-16) = 0$

$x^2+1=0$	$x^2-16=0$
$x^2=-1$	$x^2=16$
Not real number (Complex numbers)	$\boxed{x=\pm 4}$

$y = \frac{4}{x}$

$x=4$	$y=1$
$x=-4$	$y=-1$

4, 1 and −4, −1

Rationalize the deno:

1) $$\frac{2}{\sqrt{10}} = \frac{2 \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}} = \frac{2\sqrt{10}}{\sqrt{100}} = \frac{\cancel{2}\sqrt{10}}{\cancel{10}_{5}} = \boxed{\frac{\sqrt{10}}{5}}$$

2) $$\frac{2}{\sqrt[3]{10}} = \frac{2 \cdot \sqrt[3]{10^2}}{\sqrt[3]{10} \cdot \sqrt[3]{10^2}} = \frac{2\sqrt[3]{100}}{\sqrt[3]{1000}} = \frac{\cancel{2}\sqrt[3]{100}}{\cancel{10}_{5}} = \boxed{\frac{\sqrt[3]{100}}{5}}$$

$10^1 \cdot 10^{\boxed{2}} = 10^3$

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Rationalize the deno.:

1) $\dfrac{2x}{\sqrt{2x}} = \dfrac{2x\cdot\sqrt{2x}}{\sqrt{2x}\cdot\sqrt{2x}} = \dfrac{2x\sqrt{2x}}{\sqrt{4x^2}} = \dfrac{\cancel{2x}\sqrt{2x}}{\cancel{2x}} = \boxed{\sqrt{2x}}$

2) $\dfrac{2x}{\sqrt[3]{4x}} = \dfrac{2x\cdot\sqrt[3]{2x^2}}{\sqrt[3]{4x}\cdot\sqrt[3]{2x^2}} = \dfrac{2x\sqrt[3]{2x^2}}{\sqrt[3]{8x^3}}$

$4x = 2^2\cdot x^1$

$= \dfrac{\cancel{2x}\sqrt[3]{2x^2}}{\cancel{2x}} = \boxed{\sqrt[3]{2x^2}}$

Rationalize the deno:

$\dfrac{1}{\sqrt[5]{x^2y^3}} = \dfrac{1\cdot\sqrt[5]{x^3y^2}}{\sqrt[5]{x^2y^3}\cdot\sqrt[5]{x^3y^2}} = \dfrac{\sqrt[5]{x^3y^2}}{\sqrt[5]{x^5y^5}}$

$= \boxed{\dfrac{\sqrt[5]{x^3y^2}}{xy}}$

Rationalize the deno:

$\dfrac{2x}{\sqrt[7]{8x^5}} = \dfrac{2x\cdot\sqrt[7]{2^4x^2}}{\sqrt[7]{2^3x^5}\cdot\sqrt[7]{2^4x^2}} = \dfrac{2x\sqrt[7]{16x^2}}{\sqrt[7]{2^7x^7}}$

Hint: $8 = 2^3$

$= \dfrac{\cancel{2x}\sqrt[7]{16x^2}}{\cancel{2x}}$

$= \boxed{\sqrt[7]{16x^2}}$

Rationalize the deno.:

$$\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{18}-\sqrt{12}}{\sqrt{9}-\cancel{\sqrt{6}}+\cancel{\sqrt{6}}-\sqrt{4}}$$

$3 - 2$

1

$$= \sqrt{9}\sqrt{2} - \sqrt{4}\sqrt{3}$$

$$= \boxed{3\sqrt{2}-2\sqrt{3}}$$

Divide: $\dfrac{-5i}{2-i}$

$$= \frac{-5i(2+i)}{(2-i)(2+i)} = \frac{-5i(2+i)}{4+\cancel{2i}-\cancel{2i}-i^2} = \frac{-\cancel{5}i(2+i)}{\underbrace{4-(-1)}_{\cancel{5}}}$$

$$= -i(2+i)$$

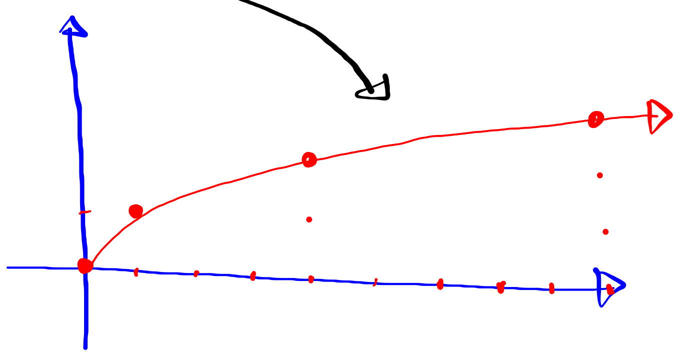
$$= -2i - i^2$$

$$= -2i-(-1)$$

$$= -2i+1 = \boxed{1-2i}$$

$f(x) = \sqrt{x}$

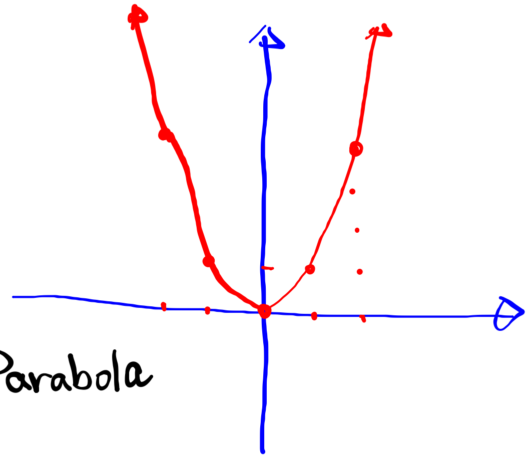
x	$y = f(x)$
0	0
1	1
4	2
9	3



$f(x) = x^2$

x	y
0	0
1	1
2	4
3	9

x	y
-1	1
-2	4
-3	9

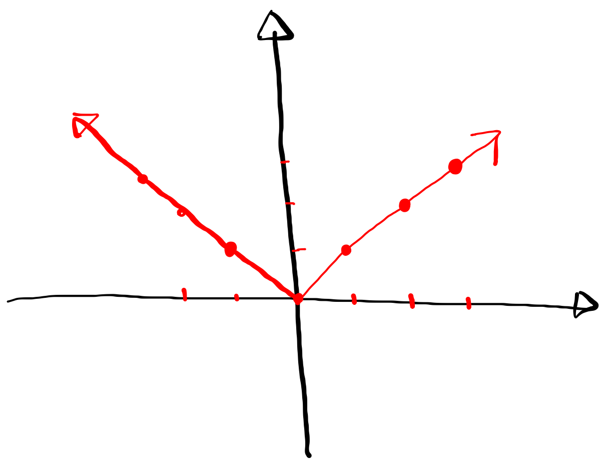


Parabola

$f(x) = |x|$

x	y
0	0
1	1
2	2
3	3

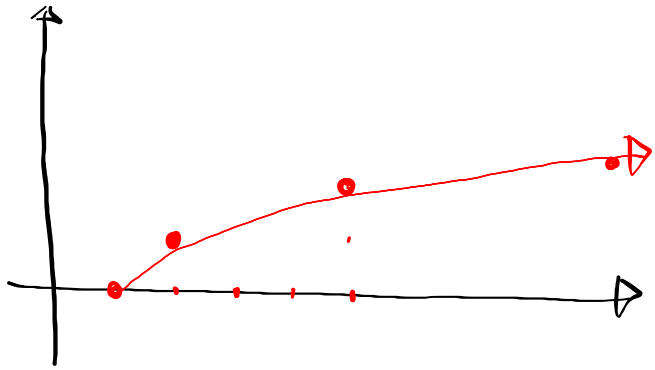
x	y
-1	1
-2	2
-3	3



Graph

$f(x) = \sqrt{x-1}$

x	y
1	0
2	1
5	2
10	3

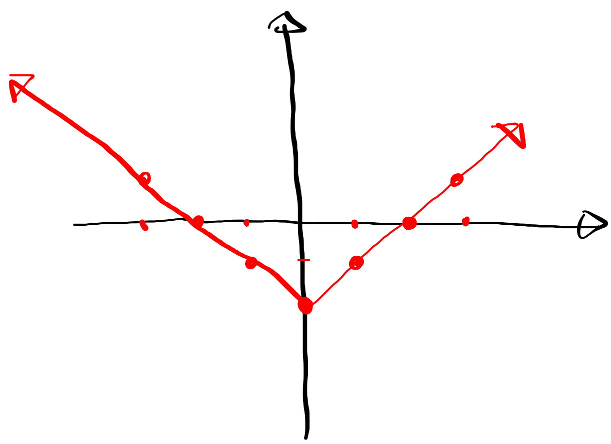


Graph

$f(x) = |x| - 2$

x	y
0	-2
1	-1
2	0
3	1

x	y
-1	-1
-2	0
-3	1



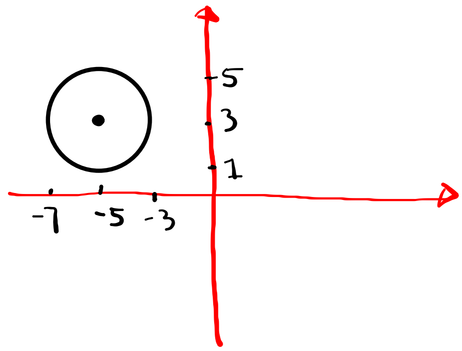
Class QZ 34

Given $(x+5)^2+(y-3)^2=4$

Center $(-5,3)$

Radius $r=2$

Draw



Domain $[-7,-3]$

Range $[1,5]$

x-Int
None
y-Int.